Backtesting Longevity Models: An International Perspective

Eric Jondeau and **Michael Rockinger**

HEC Lausanne

September 1, 2016

Think-Tank 2016, HEC

Organisation

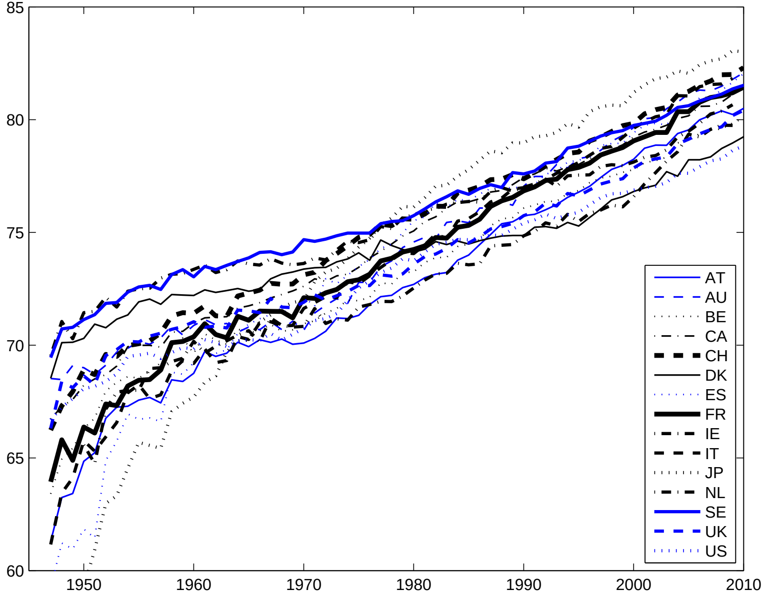
- Give an idea what longevity is about
- Tell you about popular mortality models
- Discuss empirical results

- We consider 15 countries
- Human Mortality Database
  - For some countries hundreds of years data. We start in 1947.
  - Data for ‘oldest old’
  - For Germany data too short
  - Use France for illustrations

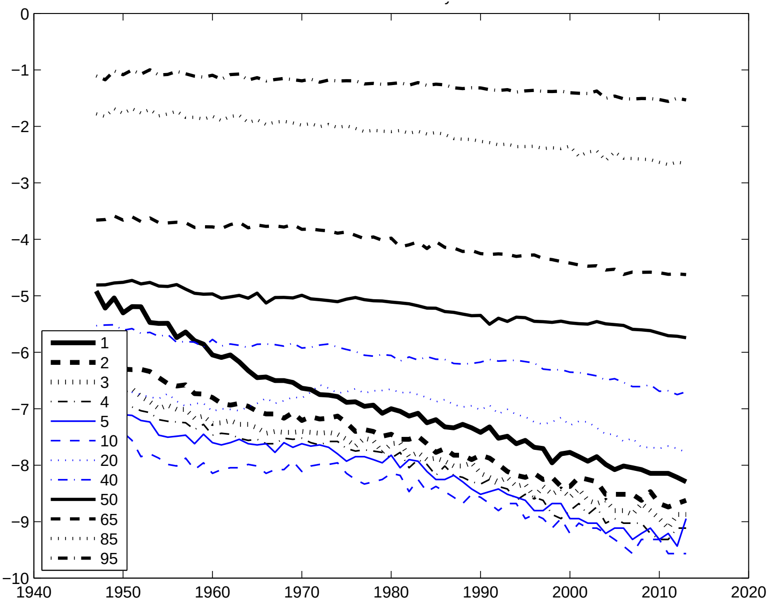
Part I

- Where is the 'problem'

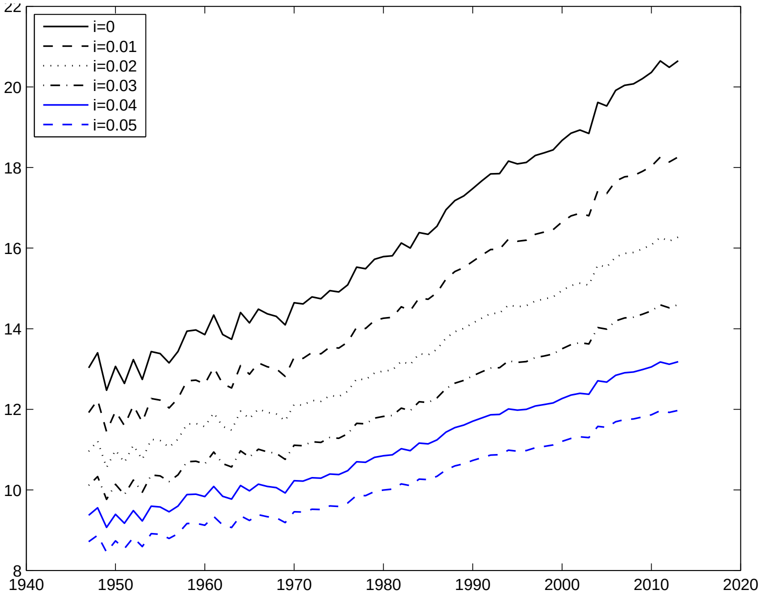
Life Expectancy, both genders at birth



log-Mortality rate, France, both genders



Annuity computations: Life Annuities France, $x = 65$



Part II

- Modeling Mortality
  - Gompertz (1825), Makeham (1860), Heligman-Pollard (1990) Param.
  - Lee and Carter (1992) Generalized Linear Models
  - Renshaw and Haberman (2006) Cohorts matter
  - Cairns, Blake, and Dowd (2006) Stochastic models

List of alternative models

LC1: $\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \varepsilon_{x,t}, \quad \varepsilon_{x,t} \sim$ Normal

LC2: $\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \varepsilon_{x,t}, \quad \varepsilon_{x,t} \sim$ Poisson

RH: $\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_{t-x}^{(3)} + \varepsilon_{x,t}$

CBD1: $\text{logit}\, q_{x,t} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \varepsilon_{x,t}$

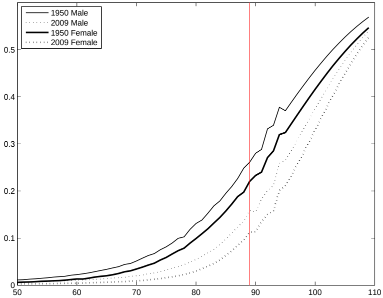
CBD2: $\text{logit}\, q_{x,t} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)} + \varepsilon_{x,t}$

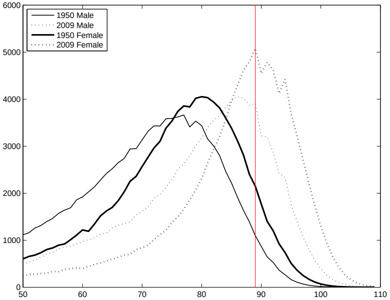
CBD3: $\text{logit}\, q_{x,t} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)}\left((x - \bar{x})^2 - \hat{\sigma}_x^2\right) + \gamma_{t-x}^{(4)} + \varepsilon_{x,t}$

A MATLAB toolbox `https://github.com/mrockinger/Matlab-Longevity-Toolbox`

Input For Estimations

Mortality and actual death count



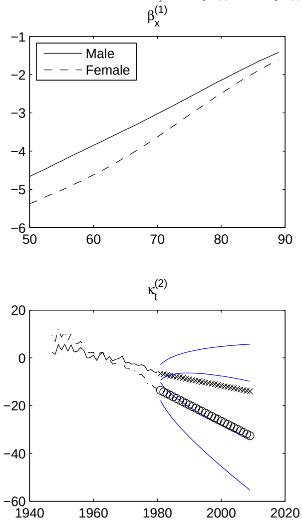


Part III

- Model estimates
- In-sample fit
- Various backtesting exercises

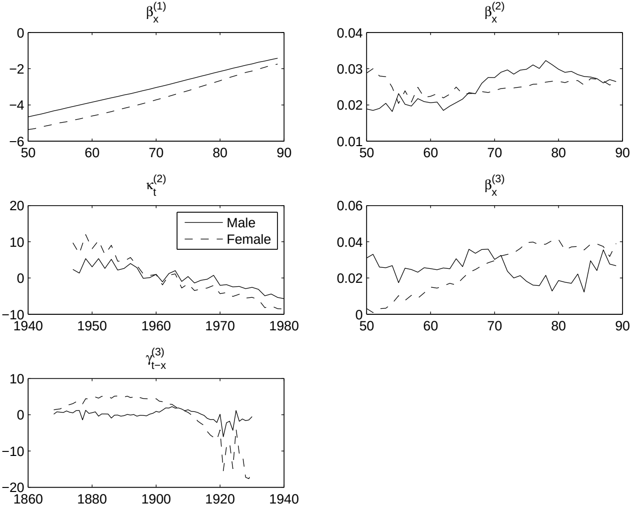
Lee and Carter, France, 1947-1980

LC2: $\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \varepsilon_{x,t}, \quad \varepsilon_{x,t} \sim$ Poisson



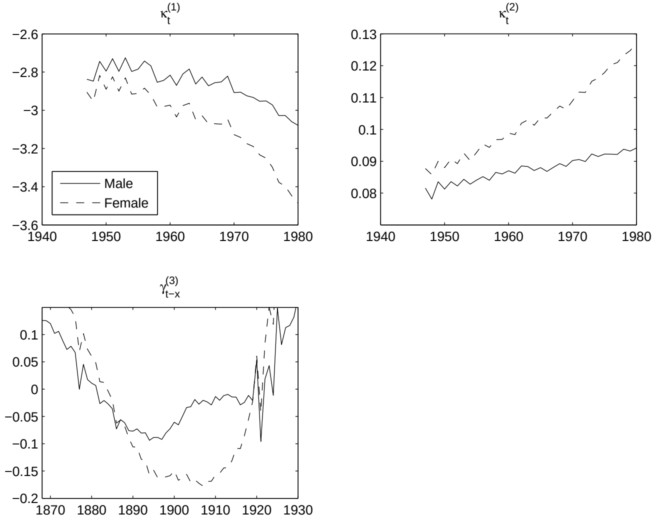
Renshaw and Haberman, France, 1947-1980

RH: $\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_{t-x}^{(3)} + \varepsilon_{x,t}$



Cairns, Blake, and Dowd, France, 1947-1980

$$\text{CBD2:} \quad \text{logit}\, q_{x,t} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)} + \varepsilon_{x,t}$$



Estimation 1947-2009, $x = 50 : 89$, In-sample RMSE

Table 2: In-sample model fit for various models. Model parameters are estimated for a sample starting in 1947 and ending in 2009 for all individuals with ages between 50 and 89. Focus is on RMSE of mortality rate and MAPE of death count.

ctry	LC1		LC2		RH		CBD1		CBD2		CBD3	
	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
Panel A: Root Mean Squared Error of Mortality Rate												
AT	0.0042	0.0034	0.0042	0.0029	**0.0036**	**0.0024**	0.0109	0.0076	0.0091	0.0482	**0.0112**	**0.1410**
AU	0.0037	0.0024	0.0036	0.0023	**0.0031**	**0.0020**	0.0073	0.0060	0.0196	0.0237	**0.0234**	**0.0498**
BE	0.0044	0.0030	0.0044	0.0025	**0.0031**	**0.0019**	0.0104	0.0070	0.0053	0.0448	**0.0114**	**0.0895**
CA	0.0036	0.0021	0.0035	0.0018	**0.0020**	**0.0016**	0.0069	0.0050	0.0040	0.0141	**0.0160**	**0.0237**
CH	0.0045	0.0036	0.0046	0.0031	**0.0038**	**0.0026**	0.0097	0.0070	**0.0151**	0.0424	0.0069	**0.1053**
DK	0.0063	0.0041	0.0061	0.0034	**0.0036**	**0.0029**	**0.0085**	0.0073	0.0044	0.0144	0.0069	**0.0796**
ES	0.0038	0.0040	**0.0035**	0.0035	0.0040	**0.0021**	0.0085	0.0063	0.0214	0.0672	**0.0305**	**0.1374**
FR	0.0030	0.0026	0.0031	0.0020	**0.0019**	**0.0011**	0.0119	0.0070	**0.0230**	0.0463	0.0220	**0.1085**
IE	0.0061	0.0046	0.0059	0.0045	**0.0054**	**0.0041**	**0.0108**	0.0088	0.0085	0.0240	0.0106	**0.0978**
IT	0.0042	0.0026	0.0042	0.0022	**0.0022**	**0.0013**	0.0098	0.0058	0.0076	0.0395	**0.0197**	**0.1291**
JP	0.0054	0.0065	0.0048	0.0048	**0.0024**	**0.0014**	0.0078	0.0066	0.0227	0.0402	**0.0275**	**0.1039**
NL	0.0054	0.0030	0.0053	0.0024	**0.0022**	**0.0020**	0.0072	0.0059	0.0046	0.0441	**0.0080**	**0.0956**
SE	0.0041	0.0028	0.0040	0.0023	**0.0032**	**0.0021**	0.0082	0.0062	0.0107	0.0311	**0.0186**	**0.1185**
UK	0.0035	0.0022	0.0036	0.0017	**0.0023**	**0.0013**	0.0070	0.0054	0.0115	0.0158	**0.0178**	**0.0553**
US	0.0023	0.0019	0.0022	0.0016	**0.0012**	**0.0009**	0.0075	0.0054	0.0092	0.0113	**0.0099**	**0.0246**

- In-sample RH model is best: in 2009 $q_{65} = 1\% \pm 0.3\%$
- CBD3 worst
- RMSE better for women

Estimation 1947-2009, $x = 50 : 89$, In-sample MAPE

- Death count

$$MAPE = \frac{1}{n_y} \sum_{t=t_1}^{t_{n_y}} \frac{1}{n_a} \sum_{x=x_1}^{x_{n_a}} \left| \frac{\hat{\xi}_{x,t} - \xi_{x,t}}{\xi_{x,t}} \right|$$

- Cross-country average

model	male	female
RH	2.5%	2.9%
LC	3.8%	4.2%
CBD1	4.5%	7.5%

- Death count
  - For 10‘000 men, $x = 65$, in 2009 observe about 100 deaths
  - RH makes an error of about 2.5 individuals.

Estimation 1947-1980, $x = 50 : 89$, Out-of-sample

- Forecast 1981-2009
- Best models are country specific
- Cross-country average

	RMSE		MAPE	
	male	female	male	female
RH	**0.97%**	**0.60%**	**21.7%**	**11.7%**
CBD1	**0.97%**	**0.61%**	**21.3%**	**13.0%**

- Death count
  - For 10‘000 lives, CBD1 error of about ±21 men and ± 13 women

Longevity Derivatives Pricing

- longevity bonds
- longevity swaps
- q-forwards
  - Instead of pricing, which is hard ☹, consider forecast errors
  - Tables are in paper
  - Here comment estimates

q-forwards out-of-sample average payoffs

- Forecast models, increasing windows, 1947-1980, 81... Horizon H
- Root mean square payoff over windows
- Consider groups of 1'000 individuals of given age x
- Compare model forecasted counts with actual death counts
- Compare period table projections with actual counts
- Average over countries

	$x = 65, H = 5$		$x = 75, H = 5$		$x = 65, H = 10$	
	male	female	male	female	male	female
LC1	9.6	6.3	26.7	21.5	25.1	16.7
LC2	9.6	6.4	27.0	21.8	25.0	16.8
RH	10.5	5.9	28.6	21.6	26.9	16.1
CBD1	**10.0**	**5.2**	**27.6**	**19.5**	**25.7**	**15.6**
CBD2	11.3	3.0	31.2	8.2	27.7	8.8
CBD3	11.7	4.0	28.5	22.1	27.5	14.7
Actuarial	**3.8**	**1.8**	**7.8**	**5.3**	**10.6**	**5.6**

Conclusion

- In a low interest rate environment with increasing length of lives longevity-risk matters more and more
- Good models in-sample not necessarily good out-of-sample. Simplicity seems best
- Even best current models in short-term unable to beat periodic-table approach. CBD1 our preferred model
- Further research in modeling mortality is necessary
    - Possible road to go: use actual causes of death
        - WHO: The estimated 1.5 million people dying from HIV globally in 2013 were 35% fewer than ... in 2005 [thanks to ART]